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Modeling traffic on crossroads

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Abstract

A simplified traffic model is studied, consisting of two vehicles traveling through a sequence of crossroads regulated by yield signs. A car approaching a yield sign stops if, in the other street, there is a car closer than a certain distance x_{tol} from the intersection. It is shown that the function which maps the state of the vehicles displays a period doubling transition to chaos. An interesting feature of the dynamics is that for extremely cautious drivers (x_{tol} too large), the map turns chaotic, thus becoming a potential source of emergent jams. Complex behavior such as the one observed in this simple system seems to be an essential ingredient in traffic patterns, and could be of relevance in studying actual crossroads situations.

Keywords: Vehicular dynamics; Crossroads; Traffic jams; Chaos

1. Introduction

Efficient transportation systems are essential for the every day activities of modern industrialized societies. This is a highly nontrivial problem, which has been analyzed using various strategies (see for example Ref. [1], and references therein). Early attempts were based on statistical and dynamical models like gas-kinetic models, car-following models, and fluid dynamic models (see, e.g., Refs. [2,3] and references therein). Modern approaches, on the other hand, are in general based on statistical methods [4], cellular automata [5–7] and computer simulations [8–12].

Problems studied include traffic jams, pedestrian flows, bus-route, etc. [13–17]. An important component of modern city traffic is road signs. For instance, traffic lights optimization has been the subject of several papers, based on cellular automata models [18], optimal velocity models [19], etc. However, not much attention has been devoted to other road signs, such as yield and stop signs, and it is the purpose of this paper to study car interactions in their neighborhood.

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Studies have revealed that complex behavior is a strong component of traffic systems [11,20]. For example, small time differences can affect traffic over long distances, or the existence of pedestrian-controlled traffic lights can randomize an otherwise synchronized traffic flow, etc.

Toledo et al. [11] considered a model consisting of a single vehicle traveling through a sequence of equidistant traffic lights. This is equivalent to a single vehicle traveling in a circle with one traffic light. It was shown that even in such a simple model, complex behavior (chaos, resonance, synchronization, etc.) arises. In this paper, we are interested in extending this model by considering yield signs.

In our case, a vehicle moves on its road, finding a sequence of yield signs. This is an straightforward extension of the work by Toledo et al. [11], replacing traffic lights with yield signs. In spite of its simplicity, it introduces several interesting new features in the model, as the yield sign by itself does not determine the behavior of the driver. Rather, the driver decides to stop or continue considering whether a second vehicle actually approaches the crossing. Even in that case the decision is not uniquely determined, as it also depends on psychological characteristics of the driver. Cautious drivers will make the decision to stop or not when the other car is far from the crossing, whereas the more aggressive drivers will decide when it is very close to the crossing. These features have nontrivial consequences when a detailed analysis is made of the evolution of the system, suggesting a specific mechanism for the emergence of complex dynamics in real city traffic, even under ideal conditions. The model is presented in detail in Section 2. Then, in Section 3 the complex behavior of the system is analyzed. Bifurcation maps and Lyapunov exponents are studied. Finally, in Section 4, conclusions are presented.

2. The model

In Toledo et al. [11], a single car moving through a sequence of equidistant traffic lights was studied. The system is equivalent to a car moving in a circle with a single traffic light. In this paper, we intend to study the effect of a yield sign. We therefore extend the previous work by considering a second car moving in a second road as shown in Fig. 1. Both roads intersect at a point, where flow is regulated by a yield sign. We label the car with the right of way as mobile A, and the other one as B. We are interested in studying the dynamics of B.

In principle, we could assume that B finds a sequence of yield signs, where the distance between the nth and nth $+ 1$ intersection will be $L_{n,B}$ for the mobile B. Also, at each intersection the cars with right of way could have different velocities. However, to keep the analysis simple, we will consider that yield signs are equidistant, and that mobile A always moves with velocity $v_{0,A} = v_{\max,A}$. Thus, the model is equivalent to having two cars moving in two circular roads intersecting at a single point [see Fig. 1(b)]. The position of mobile A is given by $x_A(t) = v_{\max,A}t$.

On the other hand, mobile B can be in one of four possible states: (a) accelerating with constant acceleration $a_{+,B}$, until its velocity reaches the cruising speed $v_{\max,B}$; (b) moving with constant speed $v_{\max,B}$; (c) braking with a negative, constant, acceleration $-a_{-,B}$ until it stops or begins to accelerate again; (d) at rest, waiting for mobile A to leave the crossing.

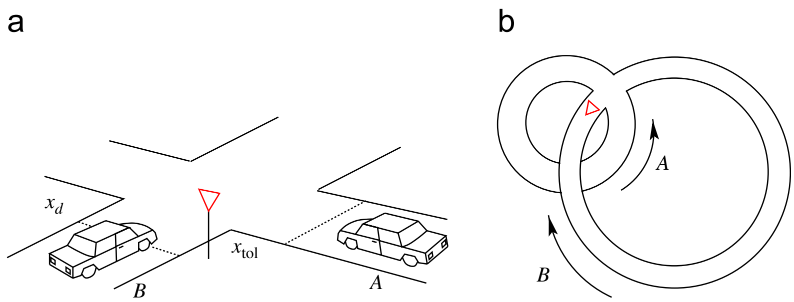

Fig. 1. (a) A sketch of the intersection to be studied: B is the mobile approaching the yield sign; A has preference to pass. x_d is the distance at which B must make a decision to stop or not. x_{tol} should be such that collisions are always avoided [see Eq. (1)]. (b) The roads are made to intersect at a periodic number of points.

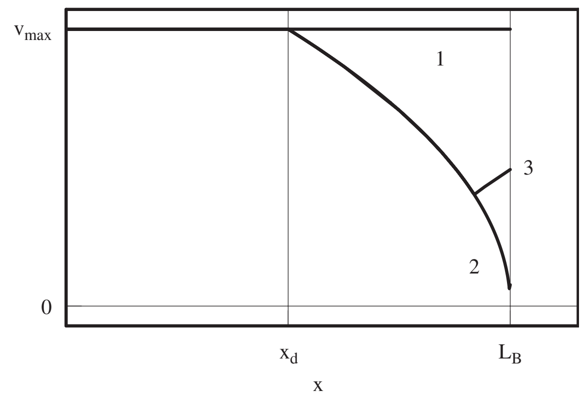

Fig. 2. Possible situations for B at the decision point x_d, namely: (1) continuing, (2) braking to stop at $x = L_B$ before A leaves the crossroad, and (3) braking and accelerating again as A leaves the crossroad before B stops completely.

To study the interaction between the cars, we introduce the tolerance parameter $x_{\rm tol}$. When B approaches the yield sign at the intersection, the driver must make a decision to stop or not at a distance $x_d = v^2_{\max,B}/2a_{-,B}$ (the last stopping distance to the yield sign) from the yield sign. If A is at a distance $x_A \leqslant x_{\rm tol}$ from the crossing, then B brakes. If A is at larger distances, B continues. $x_{\rm tol}$ depends on the position and velocity of A, but also on driver B's psychological features. Aggressive drivers will take more risks, so $x_{\rm tol}$ will be shorter. However, it is convenient to set a restriction on $x_{\rm tol}$ in order to avoid collisions. First, notice that, in our model, B always reaches the decision point x_d at maximum velocity $v_{\max,B}$. This is because positive accelerations may only occur while leaving the crossing, and the car reaches maximum velocity within a distance short compared with the distance to the next crossing; and negative accelerations can only occur at and after x_d, if B is forced to stop. On the other hand, A always moves at constant speed. If B decides to brake, then there will be no collision, given the relation between x_d and a_-. If B decides not to brake, there will be a collision only if both cars reach the crossing at the same time $t_{\rm coll}$. This occurs if A is at a distance $x_{A,c}$ from the crossing such that $t_{\rm coll} = v_{\max,A}/x_{A,c} = v_{\max,B}/x_d$, which leads to

$$x_{A,c} = \frac{v_{\max,A} v_{\max,B}}{2a_{-,B}}. \tag{1}$$

Thus, taking the restriction $x_{\rm tol} > x_{A,c}$ is enough to avoid all collisions. (Of course, it is possible to improve the estimation of $x_{A,c}$ by taking into account the streets' width and car size, for instance, but we neglect such effects in this model.)

In our model, if B decides to stop, it accelerates again as soon as A leaves the crossroad, so two things can happen to B: either it stops completely and waits until the mobile A passes, or accelerates before stopping completely. Fig. 2 shows the types of possible trajectories between two yield signs in the present model.

Applying this set of rules, we can study the evolution of the system. Its state will be characterized by the time t_n at which B reaches the nth intersection [or crosses by the nth time the single intersection in Fig. 1(b)], and by the velocity v_n at the nth intersection.

We thus determine a 2-D map $M(t_n, v_n)$ that evolves the state (t_n, v_n) at the nth crossing to the state (t_{n+1}, v_{n+1}) at the $(n+1)$th intersection. This map for the dynamics of B is constructed explicitly in the appendix, and is very similar to the map in the model of Toledo et al. [11], since A is essentially a traffic light for B. However, there is a crucial difference: the decision point $x_{\rm tol}$ is now dynamics-dependent. In the former model [11], it was fixed only by the braking capability of the car.

3. Analysis

In this work, we will consider a single maximum velocity, $v_{\max,A} = v_{\max,B} = v_{\max} = 14\,\mathrm{m/s}$. The cruise time T_c will be defined as the time it takes for a car moving with velocity $v_{\max}$ to move from one intersection to the next one. Thus, $T_{c,\{A,B\}} = L_{\{A,B\}}/v_{\max,\{A,B\}}$. We define the acceleration ratio of B as $a = a_+/a_-$. We will assume

$a_+ = 2\,\mathrm{m/s^2}$, $a_- = 6\,\mathrm{m/s^2}$, so that $a = \frac{1}{3}$. We also take $L_A = 200\,\mathrm{m}$. These parameters are consistent with average city traffic conditions.

As the car B iterates through the yield sign sequence, we can observe that complex behavior appears for certain ranges of $T_{c,B}/T_{c,A}$. Fig. 3 shows the velocity of B at each intersection, for $T_{c,B}/T_{c,A} = 0.88$ [Fig. 3(a)], and $T_{c,B}/T_{c,A} = 0.856$ [Fig. 3(b)]. A period-2 solution is clearly observed in Fig. 3(a), where B is caught in every second intersection by A, affecting the effective traffic flow, whereas in Fig. 3(b) more complex orbits appear, giving rise to nontrivial traffic flow.

As $T_{c,B}/T_{c,A}$ varies, a bifurcation diagram is obtained for the speed of mobile B. This is shown in Fig. 4(a) for $x_{\mathrm{tol}} = L_A/2$ (low risk of collision) and in Fig. 4(b) for $x_{\mathrm{tol}} = L_A/6$ (high risk). In the particular case when $T_{c,A} = T_{c,B}$, both cars are synchronized, B is not affected by A, and thus will always cross the intersection with maximum velocity. (In general, this behavior will not be seen if appropriate initial conditions are chosen, so that although $T_{c,A} = T_{c,B}$, B sees A at the intersection and thus applies the brakes. But this is a transient effect, absent when the evolution is followed for long enough times.) The bifurcation diagram of Fig. 4 suggests a period doubling bifurcation to chaos as we decrease the rate $T_{c,B}/T_{c,A}$. Also, when $T_{c,B} < T_{c,A}$ a crisis occurs for several values of $T_{c,B}/T_{c,A}$ as a function of x_{tol}, where the chaotic attractor collides with one of the velocity thresholds, producing an inverse period doubling bifurcation.

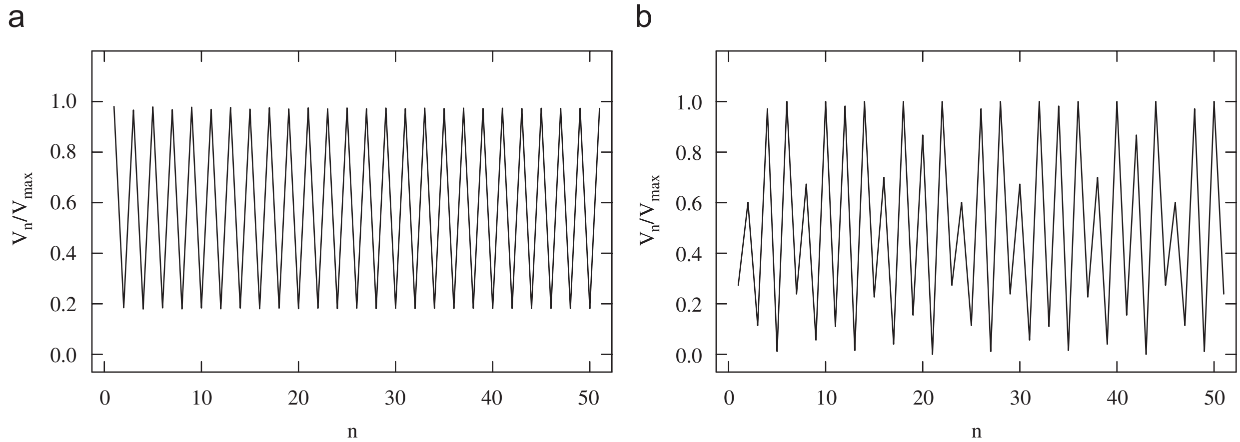

Fig. 3. The iterated map for the speed v_n at the nth crossing point, for: (a) $T_{c,B}/T_{c,A} = 0.88$ and (b) $T_{c,B}/T_{c,A} = 0.856$, for $x_{\mathrm{tol}} = L_A/2$, $L_A = 200\,\mathrm{m}$, $a_+ = 2\,\mathrm{m/s^2}$, and $a_- = 6\,\mathrm{m/s^2}$. The transient has been removed.

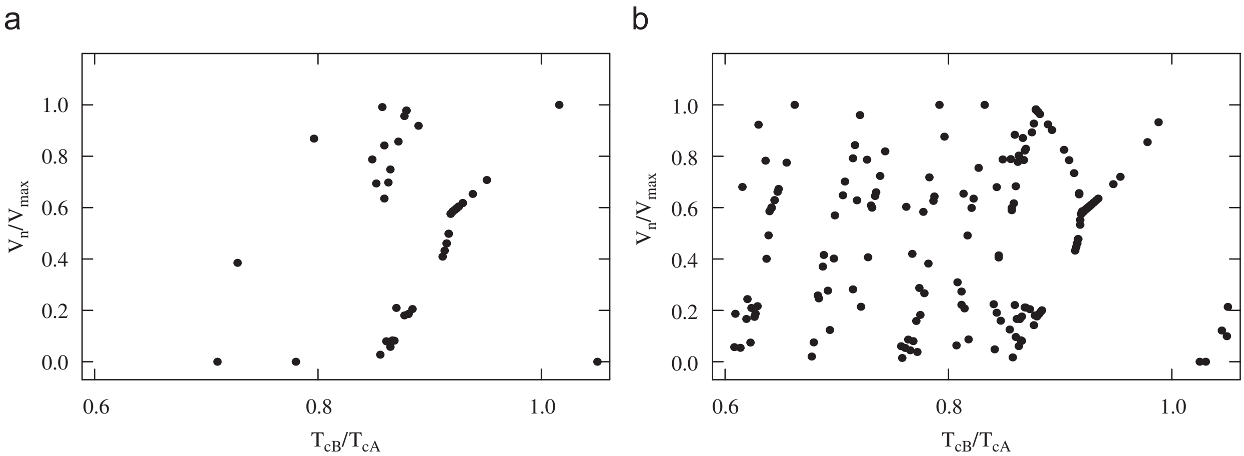

Fig. 4. The bifurcation diagram for the speed against $T_{c,B}/T_{c,A}$ for two different values of x_{tol}, (a) $x_{\mathrm{tol}} = L_A/2$ (low risk of collision) and (b) $x_{\mathrm{tol}} = L_A/6$ (high risk). The other parameters are as in Fig. 3: $L_A = 200\,\mathrm{m}$, $a_+ = 2\,\mathrm{m/s^2}$, $a_- = 6\,\mathrm{m/s^2}$. The transient has been removed.

In Fig. 4(a), we observe a behavior similar to the case studied by Toledo et al. [11]. This is expected, because $x_{\mathrm{tol}} = L_A/2$ means half of the time A is in a state that forces B to stop, and the other half in a state that allows B to continue. Thus, A can be considered as a traffic light for B, with a frequency of light change equal to $\omega = 2\pi L_A/v_{\max}$. For values $T_{c,B} > T_{c,A}$, we obtain completely different dynamics, behavior that resembles a phase transition [12]. If we zoom into one of the frequency ranges where the map displays chaotic behavior, as shown in Fig. 5(a), we find an intricate structure of periodic and chaotic behavior, as expected of a chaotic regime after a period doubling bifurcation.

Estimating the relevance of this chaotic behavior and its sensitivity to perturbation and noise, may be of importance in control strategies [21]. In this sense a finite amplitude Lyapunov exponent can be estimated [11,22]. Let us take a trajectory in the attractor that starts from a given state ($\tau_0 = t_0/T_c, u_0 = v_0/v_{\max}$) and an initially perturbed trajectory starting from ($\tau_0 + \delta_0, u_0$), with $\delta_0 \ll \tau_0$. After n iterations of the map the error becomes δ_n. Care must be taken to include only the scaling region where

$$\delta_n \sim \delta_0 \mathrm{e}^{\lambda n}.$$

Given an initial condition over the attractor an exponent can be estimated by a fitting procedure in the scaling region. Of course, the discontinuous nature of the map complicates this calculation, where for example, both trajectories can reach the same state in one step, yielding $\lambda = -\infty$. Nevertheless, a final Lyapunov exponent can still be constructed by excluding those exceptional cases. Fig. 5(b) shows the Lyapunov exponent as a function of $T_{c,B}/T_{c,A}$ and the chaotic behavior arising in this interaction ($\lambda > 0$).

The parameter x_{tol} plays a very important role in the system behavior. Fig. 6 shows the result of varying the tolerance distance, for $L_B = 172$ m and $T_{c,B}/T_{c,A} = 0.862$. For small tolerances ($x_{\mathrm{tol}} \sim L_A/10$) the behavior corresponds to low period orbits, which by increasing x_{tol} becomes an intricate pattern, seemingly chaotic. The corresponding Lyapunov exponent becomes positive for a certain range of x_{tol}. This makes sense because the cautious driver has more time to acquire any dynamical state before the intersection (see trajectory labeled 3 in Fig. 2). In this case the Lyapunov exponent is indicating the richness of its dynamics.

It is interesting to note that, in general, for different values of T_{cB}/T_{cA}, the chaotic region appears for larger values of x_{tol}, but complex behavior is highly reduced in the upper neighborhood of $x_{A,c}$. This also occurs for traffic parameters which fit well to ordinary city traffic conditions, *i.e.*, those used throughout this paper. This suggests that, even though cautious driving (larger x_{tol}) is preferable to aggressive one, extremely cautious drivers may be unfavorable to efficient traffic flow. In effect, a very large value of x_{tol} means the driver approaching the yield sign brakes even if the other car is very far from the crossing. By taking the decision to brake too early, his/her resulting evolution tends to be chaotic, which may lead to the emergence of traffic jams for cars behind him/her. Thus it is better to choose a lower x_{tol}, near the critical value $x_{A,c}$, so that even in the worst scenario, chaotic behavior would be avoided with a higher probability. We stress, though, that

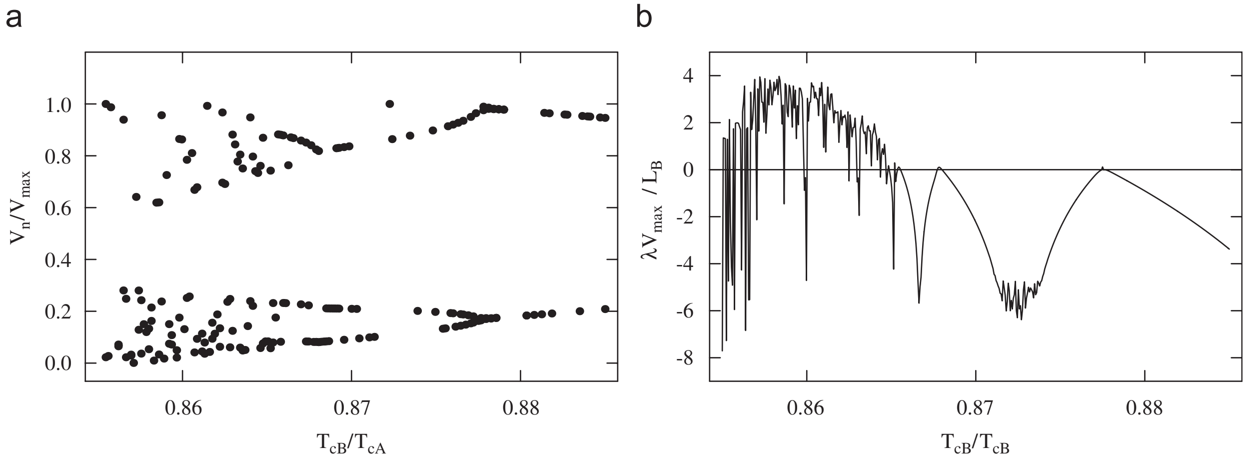

Fig. 5. The bifurcation diagram: (a) zoom of a chaotic region in Fig. 4(a), and (b) the associated Lyapunov exponent.

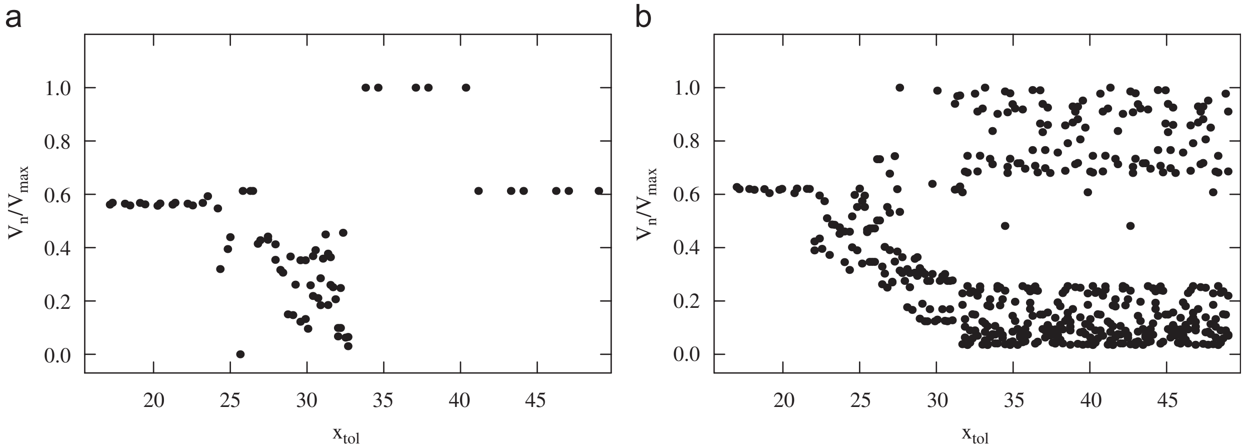

Fig. 6. The bifurcation diagram with x_{tol} for: (a) $L_B = 152\,\mathrm{m}$ ($T_{c,B}/T_{c,A} = 0.760$) and (b) $L_B = 172\,\mathrm{m}$ ($T_{c,B}/T_{c,A} = 0.862$).

avoiding (1) precludes the possibility of a collision. For Fig. 6, $x_{A,c} = 16.3\,\mathrm{m}$, so there is a range of values (up to $x_{\mathrm{tol}} \simeq 21\,\mathrm{m}$) that x_{tol} can take which both avoid collisions and chaos. Notice that these values of x_{tol} approximately correspond to 10% of the total street length ($L_B = 172\,\mathrm{m}$) in this case.

4. Conclusions

In this work, a simple model of two cars whose flow is regulated by a sequence of yield signs at intersections was studied. This can be regarded as an extension of the work of Toledo et al. [11], where a single car traveling through a sequence of traffic lights was considered. In our present model, the traffic lights are replaced by yield signs. A two dimensional-map model is derived which describes the dynamics of the system. Despite the simplicity of the model, it exhibits unpredictable behavior which suggests that chaotic behavior may be an essential part of any traffic network.

Since transients have been removed, this model applies to long trips through the city. Short trips would be better described by the transient behavior.

The analysis highlights the difficulties involved in the control of traffic flow in cities. With one car, we already have a possibly complicated situation. As we include more cars, we can only expect more interesting and complicated situations, i.e., emergent phenomena arising from several kinds of interactions. Controlling such systems will usually require a control strategy that involves a large number of interacting agents.

Several improvements are possible for the model presented here, to make it closer to real traffic situations. The cars with the right of way need not have the same velocity, and yield signs need not be equidistant. And of course, in general, not one but a flux of cars in both roads will reach the intersection. This can modify some parameters in the model. For example, the averaged cruising speed of the system $v_{\max}$, and the accelerations a_+ and a_-, may change with the number and density of interacting cars as discussed before. This will certainly affect the overall dynamics in the system.

Although our model does not pretend to describe exactly real traffic, we believe that it reflects an important feature of the system at hand, that is, the unavoidable complex behavior even at its simplest level. Moreover, it suggests us an additional possible origin of fluctuations in real traffic, and noting that they are present even under ideal conditions, we conclude that it is very necessary a complete knowledge of the basic interactions to have a hope of controlling those instabilities. In effect, spontaneous small fluctuations can lead to large emergent traffic jams [21]. But fluctuations are a consequence of an underlying rich dynamics, and that is precisely what we obtain for a cautious driver maneuvering at the crossroad. This does not mean, of course, that aggressive driving is the solution to avoid traffic jams in this system. Rather, this result suggests the need to regulate just how cautious a driver can be in order to not to contribute to block the flux.

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Appendix A. The $M(t, v)$ map

In this section we construct the exact map for the dynamical states of mobile B. The distance between two consecutive crossings and between the origin and the first crossing is L_B. Position of mobile A is given by

$$x_A(t) = v_{\max,A}t.$$

At any given point, the state of B is characterized by three variables: time t, distance x from the origin, and velocity v.

After crossing the nth yield sign, B reaches $v_{\max,B}$ at a certain time $t_{c,B}$ and a distance $x_{c,B}$ from the origin. Thus, at this point, the state of B is given by

$$\begin{aligned}
x_{c,B} &= \frac{v_{\max,B}^2 - v_{n,B}^2}{2a_{+,B}},\\
t_{c,B} &= t_{n,B} + \frac{v_{\max,B} - v_{n,B}}{a_{+,B}},\\
v_{c,B} &= v_{\max,B}.
\end{aligned}$$

Then B continues at constant velocity until it reaches the decision point x_d. This is the point where B has to brake in order to stop at the crossing (see Fig. 2). At the decision point, the state is given by

$$\begin{aligned}
x_{d,B} &= L_{n+1,B} - \frac{v_{\max,B}^2}{2a_{-,B}},\\
t_{d,B} &= t_{c,B} + \frac{x_{d,B} - x_{c,B}}{v_{\max,B}},\\
v_{d,B} &= v_{\max,B}.
\end{aligned}$$

Here we have two choices depending on the position of A. First, if $L_A - x_A(t, B) > x_{\text{tol}}$, B does not brake and reaches the $(n+1)$th yield sign with a state

$$\begin{aligned}
x_{n+1,B} &= L_{n+1,B},\\
t_{n+1,B} &= t_{d,B} + \frac{L_{n+1,B} - x_{d,B}}{v_{\max}},\\
v_{n+1,B} &= v_{\max,B}.
\end{aligned}$$

On the other hand, if $L_A - x_A(t, B) < x_{\text{tol}}$, the car must brake with acceleration $a_{-,B}$, and it will take an extra time Δt to reach the $(n+1)$th yield sign and stop, with $\Delta t = v_{\max,B}/a_{-,B}$. This time must be compared with the time it takes for A to cross the intersection,

$$t_p = t_{d,B} + \frac{L_A - x_A(t_{d,B})}{v_{\max,A}}.$$

As shown in Fig. 2, if A reaches the crossing before B stops, then B can accelerate again before reaching the yield sign. Otherwise, B stops and waits until A leaves the crossing to start again. Therefore, if $t_{d,B} + \Delta t < t_p$, B will cross the $(n+1)$th yield sign with

$$\begin{aligned}
x_{n+1,B} &= L_{n+1,B},\\
t_{n+1,B} &= t_p,\\
v_{n+1,B} &= 0.
\end{aligned}$$

On the other hand, if $t_{d,B} + \Delta t > t_p$, B starts accelerating at the state

$$x_p = x_{d,B} + v_{d,B}(t_p - t_{d,B}) - \tfrac{1}{2}a_{-,B}(t_p - t_{d,B})^2,$$
$$t_p = t_p,$$
$$v_p = v_{d,B} - a_{-,B}(t_p - t_d),$$

and again we have two cases before it reaches $L_{n+1,B}$. We need to determine if the car reaches $v_{\max,B}$ before A reaches the intersection. The distance at which B reaches $v_{\max,B}$ is

$$x_{m,B} = x_p + (v_{\max,B}^2 - v_p^2)/2a_{+,B}.$$

Therefore, if $x_{m,B} > L_B$, then B reaches the intersection with

$$x_{n+1,B} = L_{n+1,B},$$
$$t_{n+1,B} = t_p + \frac{v_{n+1,B} - v_p}{a_{+,B}},$$
$$v_{n+1,B} = \sqrt{v_p^2 + 2a_{+,B}(L_{n+1,B} - x_p)}.$$

Otherwise, it reaches $v_{\max,B}$ at

$$x_{m,B} = x_{m,B},$$
$$t_{m,B} = t_p + \frac{v_{\max,B} - v_p}{a_{+,B}},$$
$$v_{m,B} = v_{\max,B},$$

and the intersection at

$$x_{n+1,B} = L_{n+1,B},$$
$$t_{n+1,B} = t_{m,B} + \frac{L_{n,B} - x_{m,B}}{v_{\max,B}},$$
$$v_{n+1,B} = v_{\max,B}.$$

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