Hysteresis provides self-organization in a plasma model

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Abstract.
The magnetosphere is a multi-scale spatio-temporal complex dynamical system. Self-organization is a possible solution to the seemingly contradicting observation of the repeatable and coherent substorm phenomena with underlying complex behavior in the plasma sheet. Self-organization, through spatio-temporal chaos, emerges naturally in a plasma physics model with sporadic dissipation.

Keywords: Self-organization, Space Plasmas, Magnetospheric dynamics

1. Introduction

There is mounting evidence that plasmas can demonstrate very complex behavior, that includes multi-scale dynamics, emergence and self-organization, phase transitions, turbulence, spatio temporal chaos, etc. (Lu, 1995; Carreras et al., 1996; Biskamp, 2000)

In the magnetosphere, there are two seemingly contradicting observations: (a) the magnetotail plasma sheet appears to be a dynamic and turbulent region (Borovsky et al., 1997; Ohtani et al., 1998), and (b) the substorm cycle seems coherent and repeatable with identifiable distinct phases (Baker et al., 1999) and predictable geomagnetic indices (Vassiliadis et al., 1995; Valdivia et al., 1996; Valdivia et al., 1999).

We suggest that these seemingly contradicting statements may be reconciled by proposing that the plasma sheet is driven into a non-equilibrium self-organized (SO) "global" state (Chang, 1999), characterized by critical behavior with scale invariant events, self-similar spatial structure, and multifractal topology. This paradigm is in sharp contrast to the standard picture of plasma sheet transport with laminar earthward flow in a well ordered magnetic field. Instead they are more consistent with the presence of elementary transport events, probably bursty bulk flows (Baumjohan et al., 1990; Angelopoulos et al. (1992)), that are accelerated in local reconnection regions. (see Fig. 1a).

There is mounting evidence that such a SO state occurs in the magnetosphere. Consolini (1997) found a power law distribution of burst strength in the AL index. For the 2 years of the AL index shown in Fig. 1b let's use AL^2 as a rough proxy for the energy dissipation rate. Obviously this is not correct, for we don't have the conductivity nor the effective area of dissipation. Still, we computed the event distribution

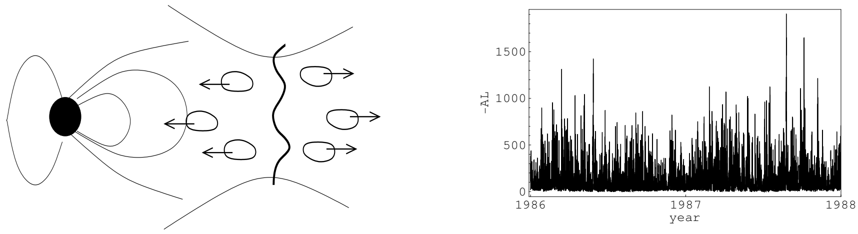

Figure 1. (a) Conceptual view of the complex magnetosphere. (b) -AL time series.

of the energy dissipated ΔE, when $AL^2 > (50nT)^2$ (see Fig. 2a). If we assume $P(\Delta E|\alpha) = \Delta E^{-\alpha}/\zeta(\alpha)$, with $\zeta(\alpha)$ as the Riemann zeta function, and apply a Bayesian argument to the measured sequence, we estimate $\alpha \sim 1.35$ from the maximum of (Goldstein et al., 2004)

$$P(\alpha|\{\Delta E\}) \sim \Pi_i P(\Delta E_i|\alpha) = e^{-\alpha \sum_i^N \ln(\Delta E_i) - N\zeta(\alpha)}$$

independently of the binning process (we assume a smooth prior $P(\alpha)$). A nonlinear exponent ξ_p with p, in the structure function

$$\left\langle \left| AL^2(t+\tau) - AL^2(t) \right|^p \right\rangle \sim \tau^{\xi_p}$$

is also a good indication of the intermittent multi-fractal dissipation in the spatio-temporal system, as suggested by Fig. 2b.

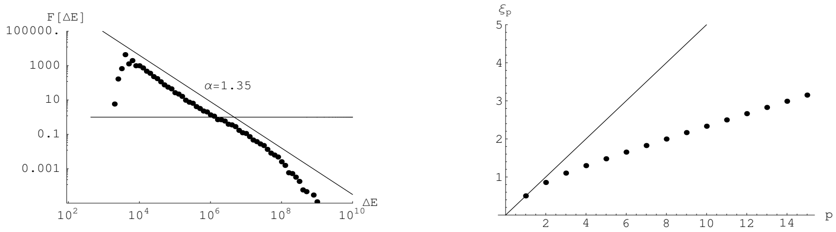

Figure 2. (a) The energy release distribution for the proxy AL^2. (b) ξ_p.

Given that we are dealing with a complex spatio-temporal system, the analysis of the single time series representation suggest just the possibility of a SO state. An analysis of the spatio-temporal ionospheric energy dissipation from Polar UVI images also found power law distributions (Lui et al., 2000; Uritsky et al., 2003). For the case of actual measurements in the tail, we can mention the work of Angelopoulos et al. (1999) that studied the nature of the intermittent properties of the BBFs. More detailed arguments in favor of this SO state can be found in Klimas et al., (2000) and Valdivia et al. (2005).

If the plasma sheet is in a SO state, then understanding self organization may be the key to understand the substorm evolution. Even though the SO state is a dynamical state in nature with a superimposed unpredictable behavior, its "global" structure is inevitable and repeatable (this is true of sandpile systems as well (Bak et al., 1987)). Thus, we are led to study substorm phenomena as an ensemble of multiscale dissipation and flow burst events in the turbulent plasma sheet under the assumption that it can reach a global SO state (see Fig. 1a).

2. Modeling

We go beyond sandpile analogues (e.g., Takalo et al., 1999) to develop plasma physics models that evolve naturally into a SO state. Take

$$\begin{aligned}
(\partial\mu/\partial t + U_j \partial\mu/\partial x_j) &= -\mu\nabla\times\mathbf{U} \\
\mu(\partial\mathbf{U}/\partial t + U_j \partial\mathbf{U}/\partial x_j) &= \mathbf{J}\times\mathbf{B} - \nabla P + \nu\nabla^2\mathbf{U} \\
(\partial P/\partial t + U_j \partial P/\partial x_j) &= -\gamma P\nabla\cdot\mathbf{U} + (\gamma-1)\mathbf{J}\cdot(\mathbf{E}+\mathbf{U}\times\mathbf{B}) - \nabla\mathbf{Q} \\
\partial\mathbf{B}/\partial t &= -\nabla\times\mathbf{E} \\
\mathbf{E}+\mathbf{U}\times\mathbf{B} &= \eta\mathbf{J} + \alpha_1\mathbf{J}\times\mathbf{B} + \alpha_2\frac{\partial\mathbf{J}}{\partial t} + \alpha_3\nabla\mathbf{P_e} + ... \qquad (1)
\end{aligned}$$

For now let $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\nabla\mathbf{Q} = 0$, and $B = \nabla\times A$. Klimas et al. (2000) and Valdivia et al.(2003) derived from the plasma equations, but using anomalous localized dissipation, a continuous 1D model of magnetic annihilation that displays self-similar event behavior, reminiscent of a SO state. Indeed, simplifying $\mathbf{A} = A_y(z,t)\widehat{y}$ we obtain

$$\frac{\partial A_y}{\partial t} = \eta\frac{\partial^2 A_y}{\partial z^2} + S(z,t) \qquad\qquad J = -\partial^2 A_y/\partial z^2 \qquad (2)$$

with $S = (\mathbf{U}\times\mathbf{B})_y$, which provides the starting point to simulate a SO state by incorporating the localized dissipation (Lu, 1995)

$$\frac{d\eta}{dt} = \frac{(q(J)-\eta)}{\tau} \qquad\qquad q(J) = \begin{bmatrix} \eta_{max} & |J| > J_c \\ \eta_{min} & |J| < \beta J_c \end{bmatrix} \qquad (3)$$

with a hysteretic trigger function q having two states. At a given position q will transition from $q = \eta_{min}$ to $q = \eta_{max}$ when $|J| > J_c$, but will not transition to the low state $q = \eta_{min}$ until $|J| < \beta J_c$ ($\beta < 1$). This 1D model displays self-similar behavior through spatio-temporal chaos for a range of conditions (Klimas et al., 2000; Valdivia et al., 2005).

As explained in Valdivia et al. (2003, 2005) this model represents the dynamics of magnetic field in the diffusion region of the magnetotail. As we add the plasma evolution in a 2D model, given by Eq. 1 but including the dynamical η, we found that the annihilated magnetic energy is transfered to the plasma in an intermittent manner generating bursts through a $J\times B$ force at the localized dissipation regions (see Valdivia et al. (2005) for details). Figure 3b displays the evolution of

Eq. 1 in 2D at some particular time t. The geometry of the system, represented in Fig. 3a, is similar to Ugai and Tsuda (1977) with a symmetric system at both $x = 0$ and $z = 0$, and an imposed constant inflow z-velocity $U_{z,o}$ and magnetic field $B_{x,o}$ at $z = \pm L_z$. We have outgoing conditions at $x = \pm L_x$ (see Valdivia et al. (2003, 2005) for more details). We can already see from Fig. 3 that even though we have strong underlying turbulence, there is a well defined global state that permits the dissipation and transport of energy through the system, but in a bursty fashion (see also Klimas et al., (2004)). This picture is very reminiscent of the behavior expected by Antonova et al., (1999).

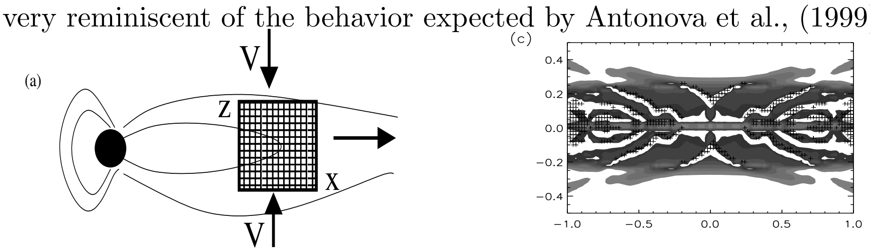

Figure 3. (a)The 2D geometry. (b) The 2D current density J at a given time. $\beta = 0.9$.

Even though we are treating the plasma sheet as a magnetofluid, the spatially dependent η brings the necessary intermittent dissipation, that is not present in regular MHD. We use this nonlinear resistivity in an attempt to characterize some of the complex microphysics behavior, with q acting like a physical current driven instability (Papadopoulos et al., 1985) with a threshold J_c that is higher than the value required to maintain the instability. Here, we are concerned with the event statistics of the collective effects of many such interacting instability sites, derived from observations and data analysis, in a complementary manner to microphysics. Furthermore, the introduction of the hysteretic loop is crucial in the generation of the loading-unloading mechanism that produces intermittent behavior, and is present in virtually all SO models including sandpiles. If we take $\beta = 1$, then η relaxes very quickly after q is turned on, hence we destroy de loading-unloading cycle. If we make $\beta << 1$ we give more time for η_{min} to smooth the spatial profile of A during the driving time, favoring a quasiperiodic evolution (Valdivia et al., 2005). It is important to stress that hysteresis is a natural phenomena that appears even in the simplest of systems, e.g., the constantly driven pendulum (Ott, 1993) which can be used as a starting point for a simulation of the charge dynamics in a slowly varying magnetic field.

Before tackling this 2D model (to be published elsewhere), we propose to go back to the 1D model which is more manageable and study its spatio-temporal chaos and multi-scale behavior.

As an illustration let's take $-L \leq z \leq L$, with $L = 20$, $\Delta x = 0.1$, $S(z) = S_o Cos(\pi z/2L)$, $J_c = 0.04$, $\eta_{max} = 5$ (normalized to $c^2 L V_a/4\pi$),

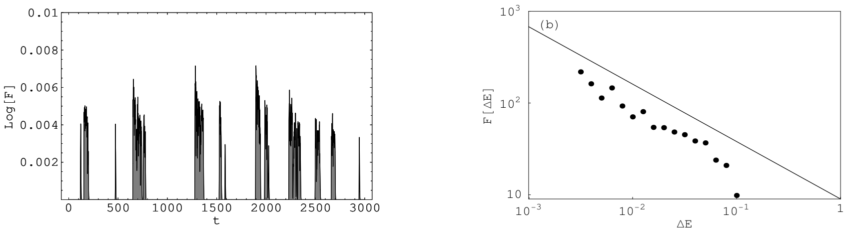

Figure 4. (a) F(t). (b) Event distribution of energy dissipated.

V_a a reference Alfven's speed, $\tau = 1$, and $J = 0$ at the boundaries. The dissipation rate $F(t) = \int \eta(x,t)J(x,t)^2 dx$ is shown in Fig. 4a for $S_o = 0.001$, $\beta = 0.9$, and $\eta_{min} = 0$. In Fig. 4b we computed the event distribution of energy dissipated, in which an event is defined for $F > F_{min} = 10^{-5}$. Using the technique discussed above, we estimated a power law index of $\alpha \approx 0.6$. Clearly, if $S_o < J_c\eta_{min}$ or $S_o > J_c\eta_{max}$ we can have a steady state solution. The bifurcation diagram with S_o, depends strongly on η_{min}, η_{max}, and β, and is illustrated in Fig. 5:

1. for $J_c\eta_{min} < S_0 < S_p$ we can have a quasiperiodic situation (see Valdivia et al., (2005) for an example using $\beta = 0.5$). This regime depends on the ratio η_{min}/η_{max} and β (Tangri et al., 2003).

2. for $S_p < S_0 < S_c$ we have a SO state, with a well defined global B_x and intermittent dissipation with self-similar statistics. As $S_0 \to S_c$ the duration of the loading and unloading cycles become the same. The time duration distribution follows a power law with $\alpha \approx 1.4$, and suggests an explanation for the distribution of BBFs observed by Angelopoulos et al., (1999).

3. for $S_c < S_0 < J_c\eta_{max}$ we have a chaotic behavior but without the loading-unloading cycle, as the separation tends to zero as $S_0 \to S_c$.

4. for $S_0 > J_c\eta_{max}$ the system responds directly to the driver. It is important not to over extrapolate, but the transition at $S_o \sim J_c\eta_{max}$ seems like a first order phase transition, and may explain the observation of Sitnov et al. (2000) and Uritsky et al. (2002).

Whether each of these behaviors is actually displayed by the magnetosphere remains to be determined, but it is suggestive to mention: (1) saw-tooth like oscillations, (2) turbulent self-similar evolution, (3) directly driven state, and (4) steady magnetospheric convection.

A singularity analysis of F is shown in Fig. 6a for the time series of Fig. 4a. Let's note that there is a clear multifractal behavior, and that it is strikingly similar to Fig. 2b. The singularity analysis can also be

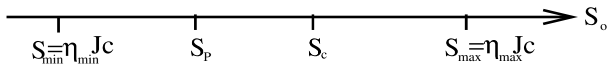

Figure 5. The phase diagram with S_0.

applied to the spatial dependence of the dissipation ηJ^2, as illustrated in Fig. 6b at three different instants during the same dissipation event.

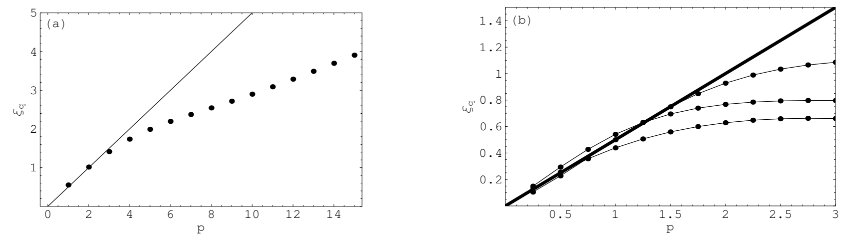

Figure 6. Singularity analysis of (a) $F(t)$ and (b) ηJ^2 in space, at three instances.

3. Conclusions and Outlook

In the magnetosphere the robust SO state is a possible solution to the seemingly contradicting observations of the repeatable and coherent substorm phenomena with underlying complex behavior in the plasma sheet. This work suggests that hysteresis may have an important role in the self-similar behavior of the magnetotail. Even though the exact details of the microphysics (ballooning, cross field current, variant of tearing, etc.) may not be accounted by the simple parameterization, it is expected that the statistical behavior of complex distributed systems is more a property of their SO state than the details of the physical processes that allow such state. This is a general characteristic of systems that are close to criticality where many systems belong to the same universality class, suggesting that it is probable that the statistics of substorms, pseudobreakups, and even the evolution of the growth and expansion phases, are unrelated to the details of the dissipation process (Shay et al., 1998) other than the dissipation allows for the establishment of a SO state. Even though, the 2D model is clearly a more appealing description of the intermittent dissipation in the magnetotail, the 1D model is more manageable and permits a more comprehensive study of the parameter space. Furthermore, some of the parameters of the model may be estimated from actual measurements, e.g., plasma sheet eddy diffusion coefficient (Borovsky et al., 1997).

The intriguing spatio-temporal multifractal chaotic behavior of the 1D model needs to be characterized in detail. For now, it is interesting to note that the system described by Eq. 3 can be discretized as an $n = L/\Delta x \geq 1$ dimensional system (the local instability size becomes a fourth relevant parameter). For $n = 1$ we have a simple nonlinear oscillator, and as we increase n, the system can become spatio-temporal chaotic through a nonstandard transition (Ott, 1993), that needs to be studied in detail. Finally, the bifurcation diagram will become even more interesting as the driver $S(t)$ is made stochastic.

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